# The life and times of Leonhard Euler

Justin Smith

# Portrait

# The early years

Born in Basel, Switzerland in 1707 to Paul Euler and Marguerite Brucker.

# Childhood

Soon after Leonhard's birth, the Euler's moved to the nearby town of Riehen, where Leonhard spent most of his childhood.

The Eulers were friends of the Bernoullis and, in particular Johann Bernoulli.

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- In 1726, he received his doctorate. His thesis was entitled *De Sono*.

# The Imperial Academy.

Peter the Great had established the Imperial Russian Academy of Sciences.

The academy provided excellent financial resources, a good library, and minimal teaching duties.

# Arrival in St. Petersburg.

Johann Bernoulli's two sons, Daniel and Nicolas were working at the Imperial Academy.

- In 1726, Nicolas died of appendicitis after only one year there.
- After Nicolas took over his brother's position, he recommended that the post in physiology he vacated by taken over by Leonhard Euler.
- After failing to win a position in the Physics Department at the University of Basel, he accepted.

# First Russian Period

- Euler spent more than a month traveling to St. Petersburg. He traveled down the Rhine by boat, crossed the German states by post wagon, then by boat from Lübeck arriving in St Petersburg on 17 May 1727.
- He lodged with Daniel Bernoulli, learned Russian, and adjusted to life in St. Petersburg.
- Unfortunately, the academy's benefactress, Catherine I, died the day of Euler's arrival.
- This led to funding cuts in the Academy. Euler served as a medical lieutenant in the Russian Navy from 1727 to 1730.

# People

At the academy, Euler met many of Europe's most prominent scientists:

- analyst, geometer Jakob Hermann, a relative;
- Daniel Bernoulli, with whom Euler was connected in the field of applied mathematics;
- Christian Goldbach, with whom Euler discussed numerous problems of analysis and the theory of numbers;
- F Maier, working in trigonometry;
- the astronomer and geographer J-N Delisle.

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- special functions

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In 1740, he shared the Paris Prize with Daniel Bernoulli and Colin Maclaurin for his work on tides.

# Calculus

Euler invented the notion of a mathematical function, and was the first to use the notation $f(x)$.
He discovered the power-series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

and

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

and Euler's Formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

# Basel Problem

In 1735, he used this to prove

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

thus solving a famous open question in number theory.
His proof:
Start with

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

and divide by $x$ to get

$$\frac{\sin(x)}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots$$

Now, note that $\sin(x)/x$ has roots at $x = n \cdot \pi$, where $n$ is a nonzero integer. So, write

$$\begin{aligned}\frac{\sin(x)}{x} &= \left(1-\frac{x}{\pi}\right)\left(1+\frac{x}{\pi}\right)\left(1-\frac{x}{2\pi}\right)\left(1+\frac{x}{2\pi}\right)\cdots \\ &= \left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{4\pi^2}\right)\cdots\end{aligned}$$

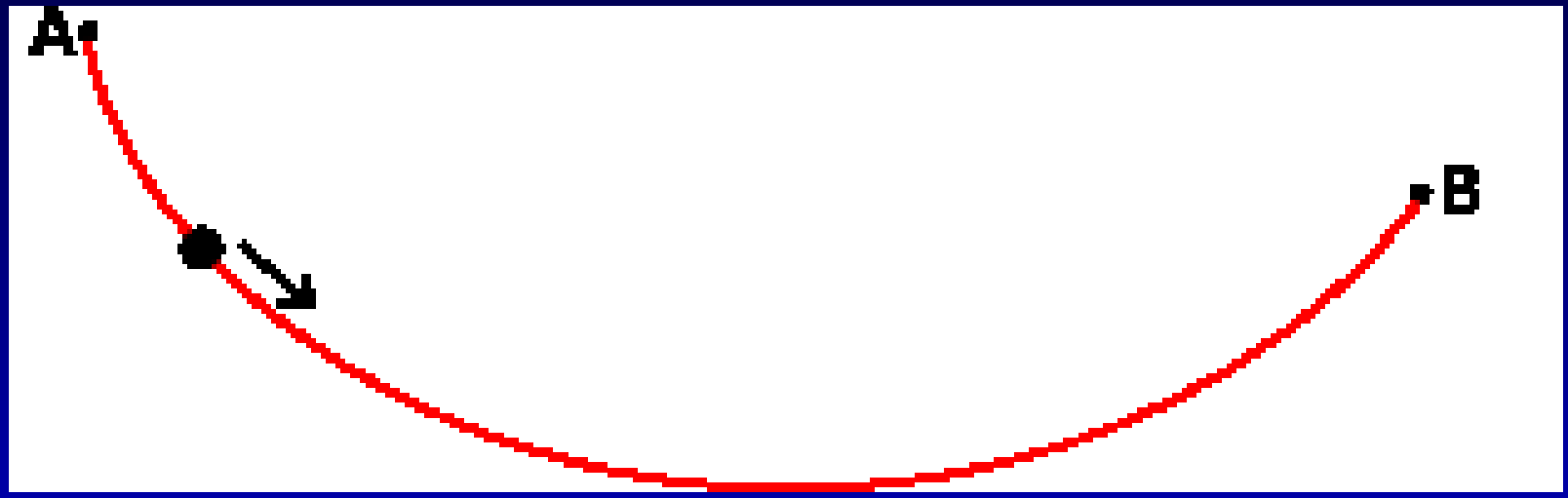
If we collect $x^2$-terms and equate them, we get

$$-\left(\frac{1}{\pi^2}+\frac{1}{4\pi^2}+\frac{1}{9\pi^2}\cdots\right) = -\frac{1}{6}$$

A hundred years later, this argument could be made rigorous.

# Brachistochrone Problem

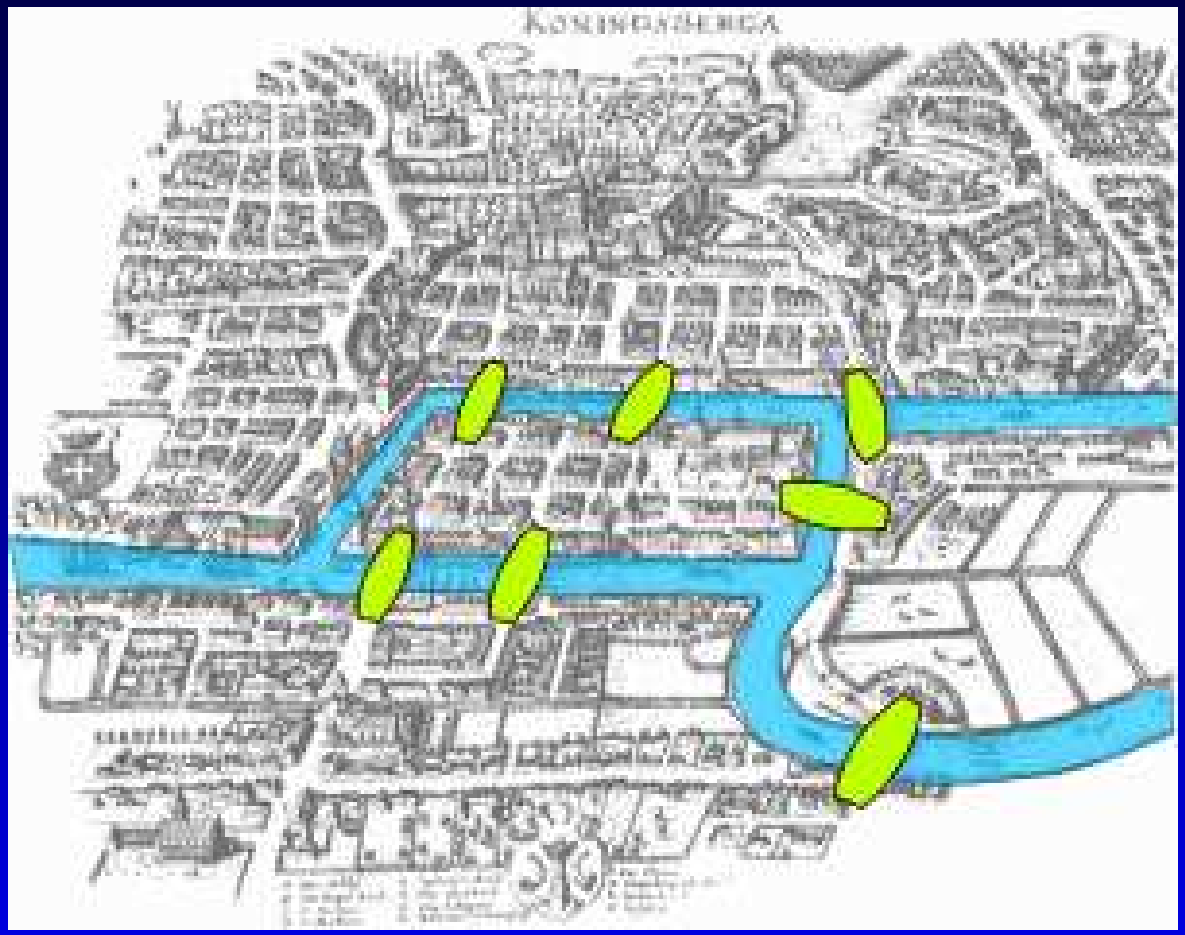
This is the problem of finding the curve with the property that a ball will roll down it in the shortest time.



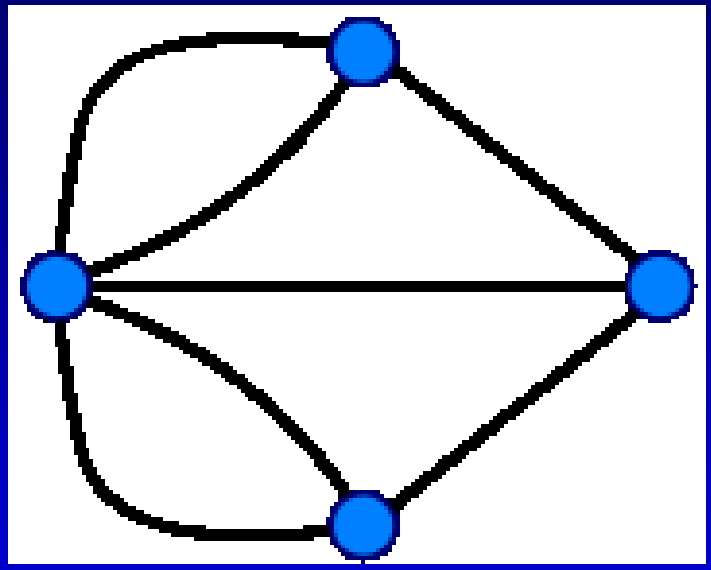

Johann Bernoulli solved it by referring to the tautochrone problem, solved by Huygens.
Euler found an analytic solution that ultimately led to the Calculus of Variations.

# Graph theory

In 1736, Euler solved the Seven Bridges of Königsburg Problem



This is generally considered to be the birth of the field of Graph Theory and its first theorem. He also introduced the notion now known at the Euler Characteristic.
Euler reduced the problem to a graph



and showed that it is impossible to traverse this in a way that includes each edge once because some nodes have odd degrees.

# Number theory

Euler defined the totient function

$$\phi(n)$$

the number of positive integers less than $n$ and relatively prime to it. Then he proved a generalization of Fermat's Little Theorem

$$a^{\phi(n)} \equiv 1 \pmod{n}$$

if $a$ is relatively prime to $n$.

# Euler's Constant

In studying the divergence of the harmonic series, Euler showed that

$$\lim_{n\to\infty} 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n$$

approached a constant, $\gamma$, called the Euler-Mascheroni constant.

# Special Functions

In attempting to interpolate the factorial function Euler defined

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

and derived a product-representation

$$\Gamma(x) = \lim_{n\to\infty} \frac{n! n^x}{x(x+1)(x+2)\cdots(x+n)}$$

# Troubles

In 1735 when given a deadline to produce the calculations to some problems, to which other mathematicians had dragged out endlessly over months, Euler focused himself and in three days the work was done — to the surprise of the Academy. This work was costly as it provoked a very high fever which placed him on death's doorstep. He was nursed back to health but not without having lost the sight in his right eye due to an abscess during his illness.

# A portrait by Emanuel Handmann

This portrayal in 1753 suggests problems of the right eye.

# The Prussian period

In 1741, the Prussian minister, Count Mardefeld proposed reviving the old Royal Society founded by Leibniz, and Frederick II (“the Great”) invited Euler to organize the operation.
Plagued by increasing political difficulties in Russian, Euler accepted and moved to Berlin.

Leibniz hall

Frederick the Great called Euler “my professor” and frequently corresponded with him from various battlefields.

# Frederick the Great

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He published his two most renowned works here: *Introductio in analysin infinitorum*, a text on functions published in 1748, and the *Institutiones calculi differentialis*, a calculus text.

# Calculus of Variations

Problem: Find the function $y(x)$ that minimizes the definite integral

$$\int_a^b F(x, y(x), y'(x))dx$$

where $F$ is some function that is given. Euler and Lagrange showed that such a function must satisfy the Euler-Lagrange equations

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0$$

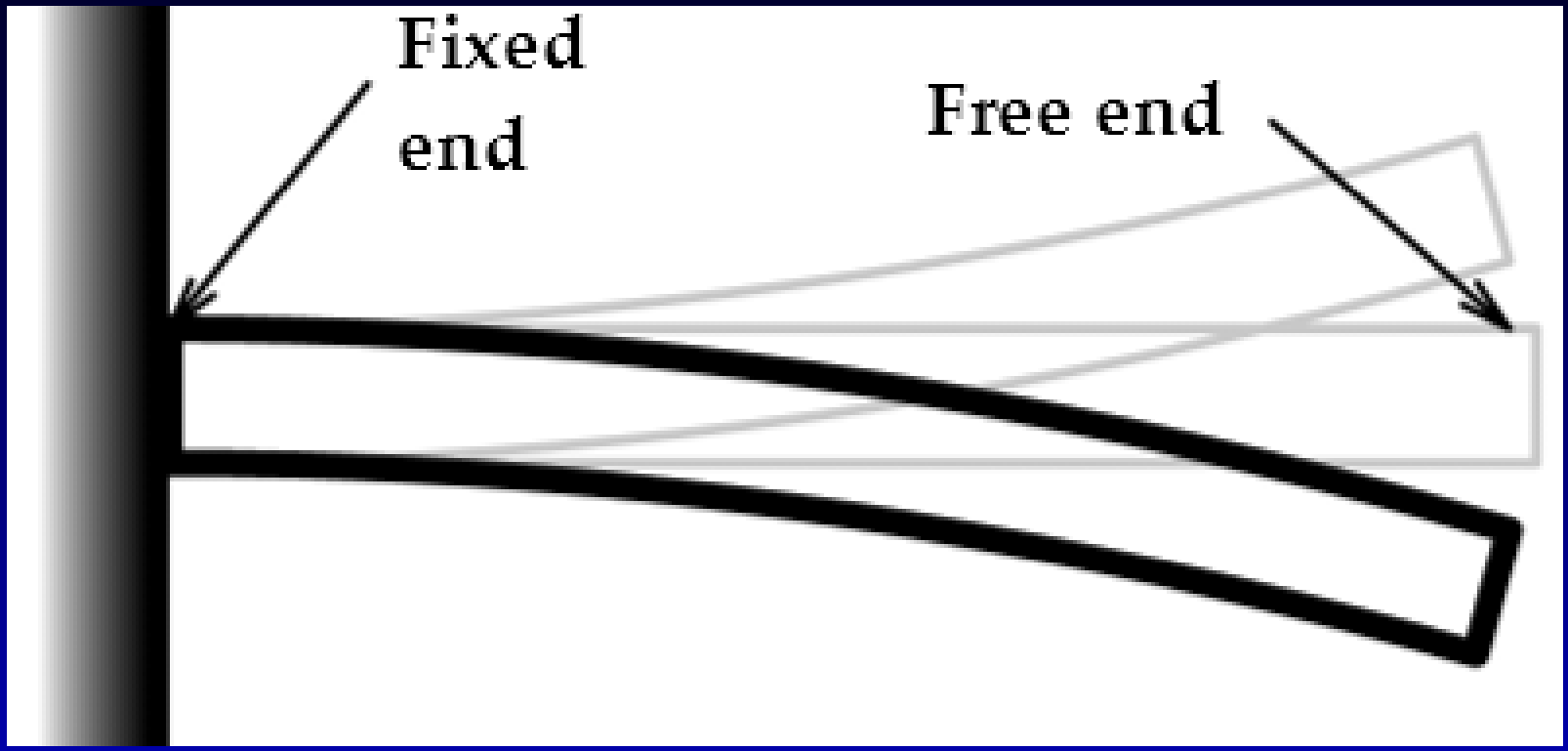
# Euler-Bernoulli Beam Equation

In 1750, Euler and Bernoulli discovered an equation for an elastic beam

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) = w$$

where:

- $u(x)$ is the deflection of a beam
- $E$ is the elastic modulus
- $I$ is the second moment of area
- $w(x)$ is the load on the beam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# Letters to a German Princess

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This book ended up being more widely read than any of his mathematical works, and was published all across Europe and in the United States.

The popularity of the *Letters* testifies to Euler's ability to communicate scientific matters effectively to a lay audience, a rare ability for a dedicated research scientist.

# San Souci

Euler was frequently invited to receptions at Frederick's San Souci Palace in Potsdam

Here, he met luminaries like Voltaire who generally regarded him as unsophisticated and socially awkward.

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Even so, his condition appeared to have little effect on his productivity, as he compensated for it with his mental calculation skills and photographic memory. For example, Euler could repeat the Virgil's Aeneid from beginning to end without hesitation.

# The War of Austrian Succession

This began after Maria Theresa of Austria succeeded her father Charles VI, Holy Roman Emperor in his Habsburg dominions in 1740, becoming Queen of Hungary and Bohemia, Archduchess of Austria, and Duchess of Parma, Piacenza, and Guastalla.

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- In 1748, the War of Austrian Succession ended with nothing really resolved.

# The Seven Years War

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- Out of respect for Euler's service in the Russian navy, the Russian army compensated him for damage to his home during the occupation.

# The Brandenburg Miracle

At the lowest point in Frederick II's reign the Tsaritsa Elizabeth died, to be succeeded by Peter III.
Peter III was strongly pro-Prussian and ordered the Russian troops to leave Berlin.

# Second Russian period

With the ascension of Catherine II (“the Great”) to the throne, conditions for the Imperial Academy greatly improved.

In 1766, Euler accepted an invitation to return to the Academy and spent the rest of his life in Russia.

His reputation in Europe was so great that the King of Poland invited him to spend ten days at his palace in Warsaw while he traveled to St. Petersburg.

A fire in 1771 cost him his home and nearly his life.

A few months later, Mr. Euler had a cataract operation by Baron de Wentzel the famous oculist and this operation returned his sight to him and to his entire family’s relief. He neglected to follow doctor’s orders and became completely blind a few days later.

His sons, the professor and the lieutenant-colonel and his colleagues, Krafft and Lexell, acted as his eyes.
In 1773, he published his *Treatise on the Construction and Steering of Ships*, developing a theory of how to minimize a ship's pitching and rolling.
He lost his wife of 40 years the same year.

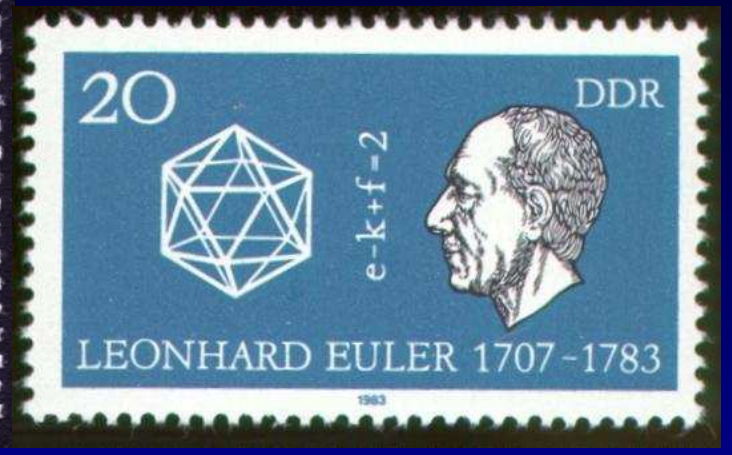
# In 1783, he died.

While having a cup of tea, he was struck with apoplexy. His last words were, “I am dying” and he lapsed into unconsciousness. He died a few hours later at the age of 76 years, 5 months and 3 days.

He’s buried in the Alexander Nevsky monastery in St. Petersburg.

At death, he left enough manuscripts to enrich the Proceedings of the Imperial Academy for 47 years.

# Commemorations


The Academy of Sciences continues to this day. The mathematics department he headed is called the Steklov Institute.